

Complex numbers in polar form

Recall that the polar form of complex numbers is $r(\cos \theta + i \sin \theta)$ where $r \in \mathbb{R}_+$ and $\theta \in [0, 2\pi)$.

Exercise 2.6

Show that if $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$ are complex numbers in polar form, then:

$$z_1 z_2 = r_1 r_2 (\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2))$$

Solution Exercise 2.6

$$z_1 = r_1(\cos \theta_1 + i \sin \theta_1) = r_1 e^{\theta_1 i}$$

$$z_2 = r_2(\cos \theta_2 + i \sin \theta_2) = r_2 e^{\theta_2 i}$$

$$\begin{aligned} z_1 z_2 &= r_1 e^{\theta_1 i} r_2 e^{\theta_2 i} = r_1 r_2 e^{(\theta_1 + \theta_2) i} \\ &= r_1 r_2 (\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)) \end{aligned}$$